

Observations on the Hyperbola

$$7x^2 - 5y^2 = 28$$

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Abstract – The hyperbola represented by the binary quadratic equation $7x^2 - 5y^2 = 28$ is analyzed for finding its non-zero distinct integer solutions. A few interesting relations among its solutions are presented. Also, knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbolas and parabolas are presented. Also, employing the solutions of the given equation, special Pythagorean triangle is constructed.

Index Terms – Binary quadratic, Hyperbola, Parabola, Pell equation, Integral solutions.

1. INTRODUCTION

The binary quadratic Diophantine equations of the form $ax^2 - by^2 = N, (a, b, c \neq 0)$ are rich in variety and have been analyzed by many mathematicians for their respective integer solutions for particular values of a, b and N . In this context, one may refer [1-16].

This communication concerns with the problem of obtaining non-zero distinct integer solutions to the binary quadratic equation given by $7x^2 - 5y^2 = 28$ representing hyperbola. A few interesting relations among its solutions are presented. Knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbolas and parabolas are presented. Also, employing the solutions of the given equation, special Pythagorean triangle is constructed.

2. METHOD OF ANALYSIS

The binary quadratic equation representing hyperbola is given by

$$7x^2 - 5y^2 = 28 \quad (1)$$

Taking,

$$\left. \begin{array}{l} x = X + 5T \\ y = X + 7T \end{array} \right\} \quad (2)$$

in (1), it reduced to the equation

$$X^2 = 35T^2 + 14 \quad (3)$$

The smallest positive integer solution (T_0, X_0) of (3) is

$$T_0 = 1, X_0 = 7$$

To obtain the other solutions of (3), consider the pellian equation

$$X^2 = 35T^2 + 1 \quad (4)$$

whose smallest positive integer solution is

$$\tilde{T}_0 = 1, \tilde{X}_0 = 6$$

The general solution $(\tilde{T}_n, \tilde{X}_n)$ of (4) is given by

$$\tilde{X}_n + \sqrt{35}\tilde{T}_n = (6 + \sqrt{35})^{n+1}, n = 0, 1, 2, \dots \quad (5)$$

Since irrational roots occur in pairs, we have

$$\tilde{X}_n - \sqrt{35}\tilde{T}_n = (6 - \sqrt{35})^{n+1}, n = 0, 1, 2, \dots \quad (6)$$

From (5) and (6) solving for $(\tilde{T}_n, \tilde{X}_n)$ we have

$$\tilde{T}_n = \frac{1}{2} \left[(6 + \sqrt{35})^{n+1} + (6 - \sqrt{35})^{n+1} \right] = \frac{1}{2} f_n$$

$$\tilde{X}_n = \frac{1}{2\sqrt{35}} \left[(6 + \sqrt{35})^{n+1} - (6 - \sqrt{35})^{n+1} \right] = \frac{1}{2\sqrt{35}} g_n$$

Applying Brahmagupta Lemma between the solutions (T_0, X_0) and $(\tilde{T}_n, \tilde{X}_n)$ the general solution (T_{n+1}, X_{n+1}) of (3) is found to be

$$T_{n+1} = X_0 \tilde{T}_n + T_0 \tilde{X}_n$$

$$X_{n+1} = X_0 \tilde{X}_n + 35T_0 \tilde{T}_n$$

$$\Rightarrow T_{n+1} = \frac{7}{2\sqrt{35}} g_n + \frac{1}{2} f_n \quad (7)$$

$$\Rightarrow X_{n+1} = \frac{7}{2}f_n + \frac{\sqrt{35}}{2}g_n \quad (8)$$

Using (7) and (8) in (2), we have

$$x_{n+1} = X_{n+1} + 5T_{n+1} = 6f_n + \sqrt{35}g_n \quad (9)$$

$$y_{n+1} = X_{n+1} + 7T_{n+1} = 7f_n + \frac{42}{\sqrt{35}}g_n \quad (10)$$

Thus (9) and (10) represent the integer solutions of hyperbola (1).

A few numerical examples are given in the following Table: 2.1 below:

Table: 2.1 NUMERICAL EXAMPLES

n	x_{n+1}	y_{n+1}
-1	12	14
0	142	168
1	1692	2002
2	20162	23856

Recurrence relations for x and y are

$$x_{n+3} - 12x_{n+2} + x_{n+1} = 0, n = -1, 0, 1, \dots$$

$$y_{n+3} - 12y_{n+2} + y_{n+1} = 0, n = -1, 0, 1, \dots$$

2.1 A few interesting relations among the solutions are given below

$$(i) 6x_{n+1} - x_{n+2} + 5y_{n+1} = 0$$

$$(ii) x_{n+1} - 6x_{n+2} + 5y_{n+2} = 0$$

$$(iii) 6x_{n+1} - 71x_{n+2} + 5y_{n+3} = 0$$

$$(iv) 71x_{n+1} - x_{n+3} + 60y_{n+1} = 0$$

$$(v) x_{n+1} - x_{n+3} + 10y_{n+2} = 0$$

$$(vi) x_{n+1} - 71x_{n+3} + 60y_{n+3} = 0$$

$$(vii) 7x_{n+1} + 6y_{n+1} - y_{n+2} = 0$$

$$(viii) 84x_{n+1} + 71y_{n+1} - y_{n+3} = 0$$

$$(ix) 7x_{n+1} + 71y_{n+2} - 6y_{n+3} = 0$$

$$(x) 71x_{n+2} - 6x_{n+3} + 5y_{n+1} = 0$$

$$(xi) 6x_{n+2} - x_{n+3} + 5y_{n+2} = 0$$

$$(xii) x_{n+2} - 6x_{n+3} + 5y_{n+3} = 0$$

$$(xiii) 7x_{n+2} + y_{n+1} - 6y_{n+2} = 0$$

$$(xiv) 14x_{n+2} + y_{n+1} - y_{n+3} = 0$$

$$(xv) 7x_{n+2} + 6y_{n+2} - y_{n+3} = 0$$

$$(xvi) 7x_{n+3} + 6y_{n+1} - 71y_{n+2} = 0$$

$$(xvii) 84x_{n+3} + y_{n+1} - 71y_{n+3} = 0$$

$$(xviii) 7x_{n+3} + y_{n+2} - 6y_{n+3} = 0$$

2.2 Each of the following expression represents a cubic integer

$$(i) (12x_{3n+3} - x_{3n+4}) + (36x_{n+1} - 3x_{n+2})$$

$$(ii) \frac{1}{12} [(143x_{3n+3} - x_{3n+5}) + (429x_{n+1} - 3x_{n+3})]$$

$$(iii) \frac{1}{7} [(42x_{3n+3} - 35y_{3n+3}) + (126x_{n+1} - 105y_{n+1})]$$

$$(iv) \frac{1}{42} [(497x_{3n+3} - 35y_{3n+4}) + (1491x_{n+1} - 105y_{n+2})]$$

$$(v) \frac{1}{497} [(5922x_{3n+3} - 35y_{3n+5}) + (17766x_{n+1} - 105y_{n+3})]$$

$$(vi) (143x_{3n+4} - 12x_{3n+5}) + (429x_{n+2} - 36x_{n+3})$$

$$(viii) \frac{1}{7} [(497x_{3n+4} - 420y_{3n+4}) + (1491x_{n+2} - 1260y_{n+2})]$$

$$(ix) \frac{1}{42} [(5922x_{3n+4} - 420y_{3n+5}) + (17766x_{n+2} - 1260y_{n+3})]$$

$$(x) \frac{1}{497} [(42x_{3n+5} - 5005y_{3n+3}) + (126x_{n+3} - 15015y_{n+1})]$$

$$(xi) \frac{1}{42} [(497x_{3n+5} - 5005y_{3n+4}) + (1491x_{3n+3} - 15015y_{n+2})]$$

$$(xii) \frac{1}{7} [(5922x_{3n+5} - 5005y_{3n+5}) + (17766x_{n+3} - 15015y_{n+3})]$$

$$(xiii) \frac{1}{49} [(-497y_{3n+3} + 42y_{3n+4}) + (-1491y_{n+1} + 126y_{n+2})]$$

$$(xiv) \frac{1}{588} [(-5922y_{3n+3} + 42y_{3n+5}) + (-17766y_{n+1} + 126y_{n+3})]$$

$$(xv) \frac{1}{49} [(-5922y_{3n+4} + 497y_{3n+5}) + (-17766y_{n+2} + 1491y_{n+3})]$$

2.3 Each of the following expression represents a bi-quadratic integer

(i) $(12x_{4n+4} - x_{4n+5}) + 4(12x_{n+1} - x_{n+2})^2 - 2$

(ii) $\frac{1}{(12)^2} [(1716x_{4n+4} - 12x_{4n+6}) + 4(143x_{n+1} - x_{n+3})^2 - 288]$

(iii) $\frac{1}{(7)^2} [(294x_{4n+4} - 245y_{4n+4}) + 4(42x_{n+1} - 35y_{n+1})^2 - 98]$

(iv) $\frac{1}{(42)^2} \left[(20874x_{4n+4} - 1470y_{4n+5}) + 4(497x_{n+1} - 35y_{n+2})^2 - 3528 \right]$

(v) $\frac{1}{(497)^2} \left[(2943234x_{4n+4} - 17395y_{4n+6}) + 4(5922x_{n+1} - 35y_{n+3})^2 - 494018 \right]$

(vi) $(143x_{4n+5} - 12x_{4n+6}) + 4(143x_{n+2} - 12x_{n+3})^2 - 2$

(vii) $\frac{1}{(42)^2} [(1764x_{4n+5} - 17640y_{4n+4}) + 4(42x_{n+2} - 420y_{n+1})^2 - 3528]$

(viii) $\frac{1}{(7)^2} [(3479x_{4n+5} - 2940y_{4n+5}) + 4(497x_{n+2} - 420y_{n+2})^2 - 98]$

(ix) $\frac{1}{(42)^2} [(248724x_{4n+5} - 17640y_{4n+6}) + 4(5922x_{n+2} - 420y_{n+3})^2 - 3528]$

(x) $\frac{1}{(497)^2} [(20874x_{4n+6} - 2487485y_{4n+4}) + 4(42x_{n+3} - 5005y_{n+1})^2 - 494018]$

(xi) $\frac{1}{(42)^2} [(20874x_{4n+6} - 210210y_{4n+5}) + 4(497x_{n+3} - 5005y_{n+2})^2 - 3528]$

(xii) $\frac{1}{(7)^2} [(41454x_{4n+6} - 35035y_{4n+6}) + 4(5922x_{n+3} - 5005y_{n+3})^2 - 98]$

(xiii) $\frac{1}{(49)^2} [(-24353y_{4n+4} + 2058y_{4n+5}) + 4(-497y_{n+1} + 42y_{n+2})^2 - 4802]$

(xiv) $\frac{1}{(588)^2} [(-3482136y_{4n+4} + 24696y_{4n+6}) + 4(-5922y_{n+1} + 42y_{n+3})^2]$

(xv) $\frac{1}{(49)^2} [(-290178y_{4n+5} + 24353y_{4n+6}) + 4(-5922y_{n+2} + 497y_{n+3})^2]$

2.4 Each of the following expression represents a nasty number

(i) $72x_{2n+2} - 6x_{2n+3} + 12$

(ii) $\frac{1}{12} [852x_{2n+2} - 6x_{2n+4} + 144]$

(iii) $\frac{1}{7} [252x_{2n+2} - 210y_{2n+2} + 84]$

(iv) $\frac{1}{42} [2982x_{2n+2} - 210y_{2n+3} + 504]$

(v) $\frac{1}{497} [35532x_{2n+2} - 210y_{2n+4} + 5964]$

(vi) $858x_{2n+3} - 72x_{2n+4} + 12$

(vii) $\frac{1}{42} [252x_{2n+3} - 2520y_{2n+2} + 504]$

(viii) $\frac{1}{7} [2982x_{2n+3} - 2520y_{2n+3} + 84]$

(ix) $\frac{1}{42} [35532x_{2n+3} - 2520y_{2n+4} + 504]$

(x) $\frac{1}{497} [252x_{2n+4} - 30030y_{2n+2} + 5964]$

(xi) $\frac{1}{42} [2982x_{2n+4} - 30030y_{2n+3} + 504]$

(xii) $\frac{1}{7} [35532x_{2n+4} - 30030y_{2n+4} + 84]$

(xiii) $\frac{1}{49} [-2982y_{2n+2} + 252y_{2n+3} + 588]$

(xiv) $\frac{1}{588} [-35532y_{2n+2} + 252y_{2n+4} + 7056]$

(xv) $\frac{1}{49} [-35532y_{2n+3} + 2982y_{2n+4} + 588]$

2.5 Each of the following expression represents a quintic integer

(i) $(12x_{5n+5} - x_{5n+6}) + 5(12x_{n+1} - x_{n+2})^3 - (60x_{n+1} - 5x_{n+2})$

(ii) $\frac{1}{(12)^3} \left[(20592x_{5n+5} - 144x_{5n+7}) + 5(143x_{n+1} - x_{n+3})^3 \right] - (102960x_{n+1} - 720x_{n+3})$

(iii) $\frac{1}{(7)^3} \left[(2058x_{5n+5} - 1715y_{5n+5}) + 5(42x_{n+1} - 35y_{n+1})^3 \right] - (10290x_{n+1} - 8575y_{n+1})$

(iv) $\frac{1}{(42)^3} \left[(876708x_{5n+5} - 61740y_{5n+6}) + 5(497x_{n+1} - 35y_{n+2})^3 \right] - (4383540x_{n+1} - 308700y_{n+2})$

$$(v) \frac{1}{(497)^3} \begin{bmatrix} (1462787298x_{5n+5} - 8645315y_{5n+7}) \\ + 5(5922x_{n+1} - 35x_{n+3})^3 \\ - (7313936490x_{n+1} - 43226575y_{n+3}) \end{bmatrix}$$

$$(vi) (143x_{5n+6} - 12x_{5n+7}) + 5(143x_{n+2} - 12x_{n+3})^3 - (715x_{n+2} - 60x_{n+3})$$

$$(vii) \frac{1}{(42)^3} \begin{bmatrix} (74088x_{5n+6} - 740880y_{5n+5}) + 5(42x_{n+2} - 420y_{n+1})^3 \\ -(370440x_{n+2} - 3704400y_{n+1}) \end{bmatrix}$$

$$(viii) \frac{1}{(7)^3} \begin{bmatrix} (24353x_{5n+6} - 20580y_{5n+6}) \\ + 5(497x_{n+2} - 420y_{n+2})^3 \\ -(121765x_{n+2} - 102900y_{n+2}) \end{bmatrix}$$

$$(ix) \frac{1}{(42)^3} \begin{bmatrix} (10446408x_{5n+6} - 740880y_{5n+7}) \\ + 5(5922x_{n+2} - 420y_{n+3})^3 \\ -(52232040x_{n+2} - 3704400y_{n+3}) \end{bmatrix}$$

$$\frac{1}{(497)^3} \begin{bmatrix} (10374378x_{5n+7} - 1236280045y_{5n+5}) \\ + 5(42x_{n+3} - 5005y_{n+1})^3 \\ -(51871890x_{n+3} - 6181400225y_{n+1}) \end{bmatrix}$$

$$(xi) \frac{1}{(42)^3} \begin{bmatrix} (876708x_{5n+7} - 8828820y_{5n+6}) \\ + 5(497x_{n+3} - 5005y_{n+2})^3 \\ -(4383540x_{n+3} - 44144100y_{n+2}) \end{bmatrix}$$

$$(xii) \frac{1}{(7)^3} \begin{bmatrix} (290178x_{5n+7} - 245245y_{5n+7}) \\ + 5(5922x_{n+3} - 5005y_{n+3})^3 \\ -(1249255x_{n+3} - 1226225y_{n+3}) \end{bmatrix}$$

$$(xiii) \frac{1}{(49)^3} \begin{bmatrix} (-1193297y_{5n+5} + 100842y_{5n+6}) \\ + 5(-497y_{n+1} + 42y_{n+2})^3 \\ -(-5966485y_{n+1} + 504210y_{n+2}) \end{bmatrix}$$

$$(xiv) \frac{1}{(588)^3} \begin{bmatrix} (-2047495968y_{5n+5} + 14521248y_{5n+7}) \\ + 5(-5922y_{n+1} + 42y_{n+3})^3 \\ -(-10237479840y_{n+1} + 72606240y_{n+3}) \end{bmatrix}$$

$$(xv) \frac{1}{(49)^3} \begin{bmatrix} (-14218722y_{5n+6} + 1193297y_{5n+7}) \\ + 5(-5922y_{n+2} + 497y_{n+3})^3 \\ -(-71093610y_{n+2} + 5966485y_{n+3}) \end{bmatrix}$$

2.6 Remarkable Observation

I. Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of hyperbola which are presented in Table: 2.6.1 below.

Table : 2.6.1 HYPERBOLAS

S.N O	HYPERBOLAS	(X_n, Y_n)
1.	$35X_n^2 - Y_n^2 = 140$	$\begin{bmatrix} (12x_{n+1} - x_{n+2}), \\ (-71x_{n+1} + 6x_{n+2}) \end{bmatrix}$
2.	$35X_n^2 - Y_n^2 = 20160$	$\begin{bmatrix} (143x_{n+1} - x_{n+3}), \\ (-846x_{n+1} + 6x_{n+3}) \end{bmatrix}$
3.	$35X_n^2 - Y_n^2 = 6860$	$\begin{bmatrix} (42x_{n+1} - 35y_{n+1}), \\ (-245x_{n+1} + 210y_{n+1}) \end{bmatrix}$
4.	$35X_n^2 - Y_n^2 = 246960$	$\begin{bmatrix} (497x_{n+1} - 35y_{n+2}), \\ (-2940x_{n+1} + 210y_{n+2}) \end{bmatrix}$
5.	$35X_n^2 - Y_n^2 = 34581260$	$\begin{bmatrix} (5922x_{n+1} - 35y_{n+3}), \\ (-35035x_{n+1} + 210y_{n+3}) \end{bmatrix}$
6.	$35X_n^2 - Y_n^2 = 140$	$\begin{bmatrix} (143x_{n+2} - 12x_{n+3}), \\ (-846x_{n+2} + 71x_{n+3}) \end{bmatrix}$
7.	$35X_n^2 - Y_n^2 = 246960$	$\begin{bmatrix} (42x_{n+2} - 420y_{n+1}), \\ (-245x_{n+2} + 2485y_{n+1}) \end{bmatrix}$
8.	$35X_n^2 - Y_n^2 = 6860$	$\begin{bmatrix} (497x_{n+2} - 420y_{n+2}), \\ (-2940x_{n+2} + 2485y_{n+2}) \end{bmatrix}$
9.	$35X_n^2 - Y_n^2 = 246960$	$\begin{bmatrix} (5922x_{n+2} - 420y_{n+3}), \\ (-35035x_{n+2} + 2485y_{n+3}) \end{bmatrix}$
10.	$35X_n^2 - Y_n^2 = 34581260$	$\begin{bmatrix} (42x_{n+3} - 5005y_{n+1}), \\ (-245x_{n+3} + 29610y_{n+1}) \end{bmatrix}$
11.	$35X_n^2 - Y_n^2 = 246960$	$\begin{bmatrix} (497x_{n+3} - 5005y_{n+2}), \\ (-2940x_{n+3} + 29610y_{n+2}) \end{bmatrix}$
12.	$35X_n^2 - Y_n^2 = 6860$	$\begin{bmatrix} (5922x_{n+3} - 5005y_{n+3}), \\ (-35035x_{n+3} + 29610y_{n+3}) \end{bmatrix}$

13.	$35X_n^2 - Y_n^2 = 336140$	$\begin{bmatrix} (-497y_{n+1} + 42y_{n+2}), \\ (2940y_{n+1} - 245y_{n+2}) \end{bmatrix}$
14.	$35X_n^2 - Y_n^2 = 48404160$	$\begin{bmatrix} (-5922y_{n+1} + 42y_{n+3}), \\ (35035y_{n+1} - 245y_{n+3}) \end{bmatrix}$
15.	$35X_n^2 - Y_n^2 = 336140$	$\begin{bmatrix} (-5922y_{n+2} + 497y_{n+3}), \\ (35035y_{n+2} - 2940y_{n+3}) \end{bmatrix}$

II. Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of parabola which are presented in Table: 2.6.2 below:

Table: 2.6.2 PARABOLAS

S.N O	PARABOLAS	(X_n, Y_n)
1.	$35X_n - Y_n^2 = 140$	$\begin{bmatrix} (12x_{2n+2} - x_{2n+3} + 2), \\ (-71x_{n+1} + 6x_{n+2}) \end{bmatrix}$
2.	$420X_n - Y_n^2 = 20160$	$\begin{bmatrix} (143x_{2n+2} - x_{2n+4} + 24), \\ (-846x_{n+1} + 6x_{n+3}) \end{bmatrix}$
3.	$245X_n - Y_n^2 = 6860$	$\begin{bmatrix} (42x_{2n+2} - 35y_{2n+2} + 14), \\ (-245x_{n+1} + 210y_{n+1}) \end{bmatrix}$
4.	$1470X_n - Y_n^2 = 246960$	$\begin{bmatrix} (497x_{2n+2} - 35y_{2n+3} + 84), \\ (-2940x_{n+1} + 210y_{n+2}) \end{bmatrix}$
5.	$17395X_n - Y_n^2 = 34581260$	$\begin{bmatrix} (5922x_{2n+2} - 35y_{2n+4} + 994), \\ (-35035x_{n+1} + 210y_{n+3}) \end{bmatrix}$
6.	$35X_n - Y_n^2 = 140$	$\begin{bmatrix} (143x_{2n+3} - 12x_{2n+4} + 2), \\ (-846x_{n+2} + 71x_{n+3}) \end{bmatrix}$
7.	$1470X_n - Y_n^2 = 246960$	$\begin{bmatrix} (42x_{2n+3} - 420y_{2n+2} + 84), \\ (-245x_{n+1} + 2485y_{n+1}) \end{bmatrix}$
8.	$245X_n - Y_n^2 = 6860$	$\begin{bmatrix} (497x_{2n+3} - 420y_{2n+3} + 14), \\ (-2940x_{n+2} + 2485y_{n+2}) \end{bmatrix}$
9.	$1470X_n - Y_n^2 = 246960$	$\begin{bmatrix} (5922x_{2n+3} - 420y_{2n+4} + 84), \\ (-35035x_{n+1} + 2485y_{n+3}) \end{bmatrix}$
10.	$17395X_n - Y_n^2 = 34581260$	$\begin{bmatrix} (42x_{2n+4} - 5005y_{2n+2} + 994), \\ (-245x_{n+3} + 29610y_{n+1}) \end{bmatrix}$

11.	$1470X_n - Y_n^2 = 246960$	$\begin{bmatrix} (497x_{2n+4} - 5005y_{2n+3} + 84), \\ (-2940x_{n+3} + 29610y_{n+2}) \end{bmatrix}$
12.	$245X_n - Y_n^2 = 6860$	$\begin{bmatrix} (5922x_{2n+4} - 5005y_{2n+4} + 14), \\ (-35035x_{n+3} + 29610y_{n+3}) \end{bmatrix}$
13.	$1715X_n - Y_n^2 = 336140$	$\begin{bmatrix} (-497y_{2n+2} + 42y_{2n+3} + 98), \\ (2940y_{n+1} - 245y_{n+2}) \end{bmatrix}$
14.	$20580X_n - Y_n^2 = 48404160$	$\begin{bmatrix} (-5922y_{2n+2} + 42y_{2n+4} + 1176), \\ (-35035y_{n+1} - 245y_{n+3}) \end{bmatrix}$
15.	$1715X_n - Y_n^2 = 336140$	$\begin{bmatrix} (-5922y_{2n+3} + 497y_{2n+4} + 98), \\ (-35035y_{n+2} - 2940y_{n+3}) \end{bmatrix}$

2.7 Generation of Pythagorean triangle

2.7.1 Let p, q be the non-zero distinct integers such that

$$p = x_{n+1} + y_{n+1}, \quad q = y_{n+1}$$

Note that $p > q > 0$ treat p, q as the generators of Pythagorean triangle $T(X, Y, Z)$ where

$$X = 2pq, \quad Y = p^2 - q^2, \quad Z = p^2 + q^2, \quad p > q > 0$$

Let A, P represents the area and perimeter of Pythagorean triangle. Then the following results are observed.

$$(i) \quad 5Y - 14X - 9Z = 56$$

$$(ii) \quad \frac{2A}{P} = x_{n+1} * y_{n+1}$$

(iii) $X + Y - \frac{4A}{P}$ is written as the sum of two squares.

(iv) $3\left(X - \frac{4A}{P}\right)$ is a Nasty number.

(v) $3(Z - Y)$ is a Nasty number.

2.7.2 Let p, q be the non-zero distinct integers such that

$$p = x_{n+1} + y_{n+1}, \quad q = y_{n+1}$$

Note that $p > q > 0$ treat p, q as the generators of Pythagorean triangle $T(X, Y, Z)$ where

$$X = 2pq, \quad Y = p^2 - q^2, \quad Z = p^2 + q^2, \quad p > q > 0$$

Let A, P represents the area and perimeter of Pythagorean triangle. In this case, the corresponding Pythagorean triangle satisfies the relation $10X - 7Y - 3Z = 56$

3. CONCLUSION

In this paper, we have presented infinitely many integer solutions for the Diophantine equations represented by hyperbola is given by $7x^2 - 5y^2 = 28$. As the binary quadratic Diophantine equations are rich in variety, one may search for the other choices of equations and determine their solutions among the suitable properties.

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